

Multisplitter Calculator

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Abstract. In this WF note, I follow up on the previous work done in WF-198. The main question I want to address is: If the RBT ratio is wrong due to slight imperfections in the plate-type, beamsplitter optics, can the ratio be corrected by adding neutral density filters (NDF) as shown in WF-198.

1. Enter measured (or desired) values of intensity ratios from first to last and NDF values

$$\text{measuredR} := \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7.1 \end{pmatrix} \quad \text{idealR} := \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix}$$

2. normalize

$$\text{RM} := \frac{\text{measuredR}}{\max(\text{measuredR})} \quad \text{RM} = \begin{pmatrix} 0.14085 \\ 0.42254 \\ 0.70423 \\ 1 \end{pmatrix} \quad \text{RI} := \frac{\text{idealR}}{\max(\text{idealR})} \quad \text{RI} = \begin{pmatrix} 0.14286 \\ 0.42857 \\ 0.71429 \\ 1 \end{pmatrix}$$

3. Using Equation 3 from WF-198 solve for x1, x2 and x3

Guess values: $x1 := 0.1$ $x2 := 0.1$ $x3 := 0.1$

Given

$$x1 \cdot x3 = \text{RM}_0 \quad \frac{x1}{x2} = \text{RM}_1 \quad \frac{x3}{x2} = \text{RM}_2$$

$$\begin{pmatrix} x1\text{val} \\ x2\text{val} \\ x3\text{val} \end{pmatrix} := \text{Find}(x1, x2, x3)$$

Results:

$$x1\text{val} = 0.2907 \quad x2\text{val} = 0.68799 \quad x3\text{val} = 0.4845$$

Check solution:

$$x1\text{val} \cdot x3\text{val} = 0.14084 \quad \frac{x1\text{val}}{x2\text{val}} = 0.42254 \quad \frac{x3\text{val}}{x2\text{val}} = 0.70423$$

$$\text{RM}_0 = 0.14085 \quad \text{RM}_1 = 0.42254 \quad \text{RM}_2 = 0.70423$$

4. Find transmission (T) and reflection (R) for each mirror

Guess values: $t1 := 0.1$ $t2 := 0.1$ $t3 := 0.1$

Given

$$\frac{t1}{1 - t1} = x1val$$

$$\frac{t2}{1 - t2} = x2val$$

$$\frac{t3}{1 - t3} = x3val$$

$$\begin{pmatrix} t1val \\ t2val \\ t3val \end{pmatrix} := \text{Find}(t1, t2, t3)$$

Results:

TRANSMISSION
T

REFLECTION
R

$$t1val = 0.22523$$

$$1 - t1val = 0.77477$$

$$t2val = 0.40758$$

$$1 - t2val = 0.59242$$

$$t3val = 0.32637$$

$$1 - t3val = 0.67363$$

5. Given the actual values of T and R can I add NDF's to get the ideal ratio back?

Guess values:

$$a := 1.0 \quad b := 1.0 \quad c := 1.0 \quad d := 1.0$$

Ideal

$$RI_0 = 0.14286 \quad RI_1 = 0.42857 \quad RI_2 = 0.71429 \quad RI_3 = 1$$

actual intensities with NDF's

$$Int := \begin{bmatrix} b \cdot c \cdot t1val \cdot t2val \cdot t3val \\ b \cdot d \cdot t1val \cdot (1 - t2val) \cdot (1 - t3val) \\ a \cdot c \cdot (1 - t1val) \cdot (1 - t2val) \cdot t3val \\ a \cdot d \cdot (1 - t1val) \cdot t2val \cdot (1 - t3val) \end{bmatrix}$$

$$\frac{Int_0}{Int_3} = 0.14084 \quad \frac{Int_1}{Int_3} = 0.42254 \quad \frac{Int_2}{Int_3} = 0.70423$$

$$1 \cdot Int_0 = 0.02996 \quad 1 \cdot Int_1 = 0.08988 \quad 1 \cdot Int_2 = 0.1498 \quad 1 \cdot Int_3 = 0.21272$$

Given

$$b \cdot c \cdot t1val \cdot t2val \cdot t3val = 0.03014$$

$$b \cdot d \cdot t1val \cdot (1 - t2val) \cdot (1 - t3val) = 0.09043$$

$$a \cdot c \cdot (1 - t1val) \cdot (1 - t2val) \cdot t3val = 0.15071$$

$$a \cdot d \cdot (1 - t1val) \cdot t2val \cdot (1 - t3val) = 0.211$$

$$\begin{pmatrix} aVal \\ bVal \\ cVal \\ dVal \end{pmatrix} := \text{Find}(a, b, c, d)$$

$$\text{Find}(a, b, c, d) = \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

Results:

$$x1val = 0.2907 \quad x2val = 0.68799 \quad x3val = 0.4845$$

Check:

$$I1 := bVal \cdot cVal \cdot t1val \cdot t2val \cdot t3val \quad I2 := bVal \cdot dVal \cdot t1val \cdot (1 - t2val) \cdot (1 - t3val)$$

$$I3 := aVal \cdot cVal \cdot (1 - t1val) \cdot (1 - t2val) \cdot t3val \quad I4 := aVal \cdot dVal \cdot (1 - t1val) \cdot t2val \cdot (1 - t3val)$$

$$I1 = 0.03051 \quad I2 = 0.09031 \quad I3 = 0.15064 \quad I4 = 0.21105$$

$$\frac{I_1}{I_4} = 0.14455$$

$$\frac{I_2}{I_4} = 0.42788$$

$$\frac{I_3}{I_4} = 0.71373$$

$$RI_0 = 0.14286$$

$$RI_1 = 0.42857$$

$$RI_2 = 0.71429$$

$$aVal = 1.0238$$

$$bVal = 1.03675$$

$$cVal = 0.98219$$

$$dVal = 0.96911$$

These are the values of NDF's needed.